and the physical background of each equation is discussed. Each chapter ends with a specific problem which is completely worked out including a program and computer time.

Specifically, in chapter one, the example of heat conduction in an insulated tapered rod with various boundary conditions is used to illustrate some methods for linear ordinary differential equations.

In chapter two, linear parabolic partial differential equations are treated. The forward difference, backward difference and Crank-Nicolson schemes are derived and the stability analysis for each is carried out.

In chapter three linear hyperbolic equations, including systems, are dealt with.

Chapter four is concerned with alternate forms of the coefficient matrices generated by the difference schemes.

Chapters five and six deal with nonlinear parabolic and hyperbolic equations, and chapter seven describes nonlinear boundary conditions.

Chapters two through seven deal with equations with one space variable.

Chapter eight describes parabolic and elliptic equations in two and three space dimensions and includes Alternating-Direction-Implicit schemes.

Chapter nine mentions some examples of complications which can arise in the problems treated earlier, for example shock waves in hyperbolic equations.

There is an appendix of algorithms useful in solving the types of equations generated by the difference schemes.

One slight drawback for use as a text is that no exercises are provided. More serious is the fact that function space approximation methods are not mentioned at all, and the title of the book might lead a student to believe that finite differences are the only known methods for obtaining numerical solutions.

STEPHEN HILBERT

Ithaca College
Ithaca, New York 14850

7[7].—J. W. WRENCH, JR., *The Converging Factor for the Modified Bessel Function of the Second Kind*, NSRDC Report 3268, Naval Ship Research and Development Center, Washington, D. C., January 1970, ii + 56 pp., 26 cm.

This report, which follows the pattern of two earlier reports [1] and [2] (see *Math. Comp.*, v. 20, 1966, pp. 457–458, RMT 71), is concerned with the development of methods and provision of auxiliary tables for the high-precision calculation of the converging factor in the asymptotic expansion of the modified Bessel function of the second kind, $K_p(x)$. While much of the analytical discussion is quite general, the practical application is restricted to functions of integer order and positive real argument.

The converging factor is defined as "that factor by which the last term of a truncated series (usually asymptotic) approximating the function must be multiplied to compensate for the omitted terms." Thus in the case of the function $K_p(x)$ we have

$$K_p(x) = (\pi/2x)^{1/2}e^{-x}\left\{1 + \frac{a_1(p)}{x} + \frac{a_2(p)}{x^2} + \cdots + \frac{a_n(p)}{x^n}\,\sigma_{n,p}(x)\right\},$$

where the $a_i(p)$ are known coefficients and $\sigma_{n,p}(x)$ is the converging factor. This may itself be expanded in a series of the form

$$\sigma_{n,p}(x) \sim \sum_{s=0}^{\infty} b_s(n, p)C_{n-p}^{(s)}(2x)(-2x)^s.$$

It is shown that the function $C_{n-p}(2x)$ is closely related to the converging factor for the probability integral, considered by Murnaghan in [2], where $C_m(m)$ is tabulated to 63D for $m = 2(1)64$.

It is advantageous to choose a value of n such that $2x$ is close to $n - p$; then, writing $m = n - p$ and $2x = m + h$, $C_m(2x)$ may be computed from the Taylor series

$$C_m(m + h) = C_m(m) + d_1(m)h + d_2(m)h^2 + \cdots .$$

An appendix to the report contains 30D values of $C_m(m)$ and its reduced derivatives $d_i(m)$ for $m = 10(1)40$. Once $C_m(2x)$ is known, its reduced derivatives $C_m^{(s)}(2x)$ can be calculated with the aid of a three-term recurrence relation, and hence the series for $\sigma_{n,p}(x)$ can be summed.

As examples of the use of the procedure described, the values of $K_0(2\pi)$ and $K_1(10)$ are evaluated to an accuracy of approximately 17D and 26D, respectively. This represents a substantial improvement on the accuracy of 10D and 14D, respectively, obtainable from the asymptotic series without the converging factor.

Attention may be drawn to an alternative method of computation; namely, the use of a Chebyshev series representation, which appears less laborious than the foregoing and permits higher accuracy to be attained over a considerably extended range of the argument. (Indeed, there is no theoretical limit to the attainable accuracy.) The corresponding coefficients are easily calculated by backward recurrence. In particular, Luke [3] has tabulated to 20D the coefficients for the auxiliary function $(2x/\pi)^{1/2}e^xK_p(x)$ as a function of the reciprocal argument $5/x$ in the range $x \geqq 5$, for $p = 0$ and 1 and several fractional values of p; in each case approximately 20S are obtainable throughout the relevant range with the use of 21 terms.

G. F. MILLER

National Physical Laboratory
Teddington, Middlesex
England

1. F. D. MURNAGHAN & J. W. WRENCH, JR., *The Converging Factor for the Exponential Integral*, DTMB Report 1535, David Taylor Model Basin, Washington, D.C., January 1963.
2. F. D. MURNAGHAN, *Evaluation of the Probability Integral to High Precision*, DTMB Report 1861, David Taylor Model Basin, Washington, D.C., July 1965.
3. Y. L. LUKE, *The Special Functions and Their Approximations*, Vol. II, Academic Press, New York, 1969.

8[9].—HANS RIESEL, *En Bok om Primtal*, Studentlitteratur, Denmark, 1968, 174 pp., (Swedish), 23 cm. Price $6.95 equivalent. (Paperback.)

This monograph on prime numbers will be of special interest to readers of *Math. Comp.* since its number theory is very close to that which appears here. In fact, many of its references are to papers that have appeared here.